Counting authorised paths in constrained control-flow graphs

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5th Bordeaux Graph Workshop

October 29th, 2019

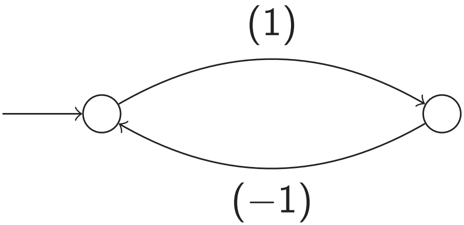
Why this question?

Model

We are given:

- A directed graph G
- A vector of dimension k initialised at 0: $V = (0, \ldots 0)$
- A start node
- For each edge e, a vector V^e of dimension k
- An integer B to bound V
- Constraints for each edge e

Avoiding infinite loops



No infinite path $\iff$ no cycle leaving v unchanged

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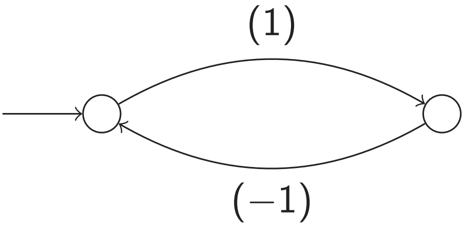

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Objectives

We look at two indicators:

- Λ : length of longest authorised path
- Π : number of distinct authorised paths

Two goals:

- Give general upper and lower bounds for Λ and Π on graphs of order n
- Compute $\Lambda(G)$ and $\Pi(G)$ for any given G

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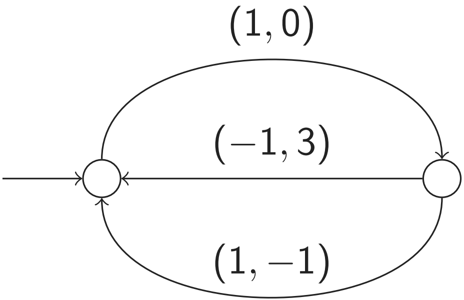
Simple example



$\Lambda = 10,\ \Pi = 14$

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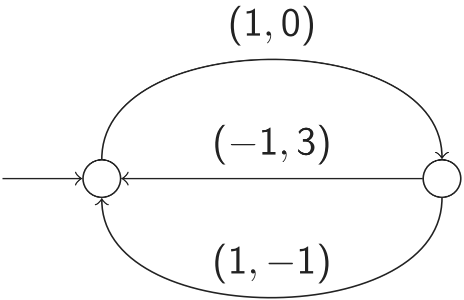

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Upper bounds

General case:

$$\Lambda \leq n \times (B+1)^k$$

Corollary:

$$\Pi \leq n^{n \times (B+1)^k}$$

When $m < k$:

$$\Lambda \leq n \times 2^m k^{m/2} \binom{k}{m} B^m$$

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Thanks to Lê Thành Dũng Nguyên:

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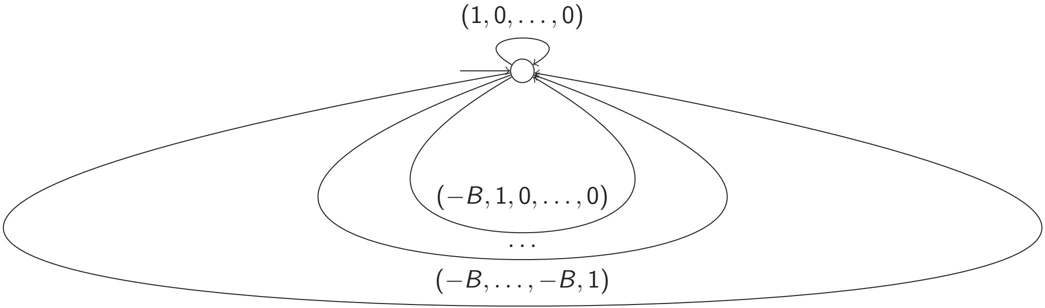
Lower bound: multigraph case



$$\Lambda = n \times (B+1)^k, \qquad \Pi = 1 + n \times (B+1)^k$$

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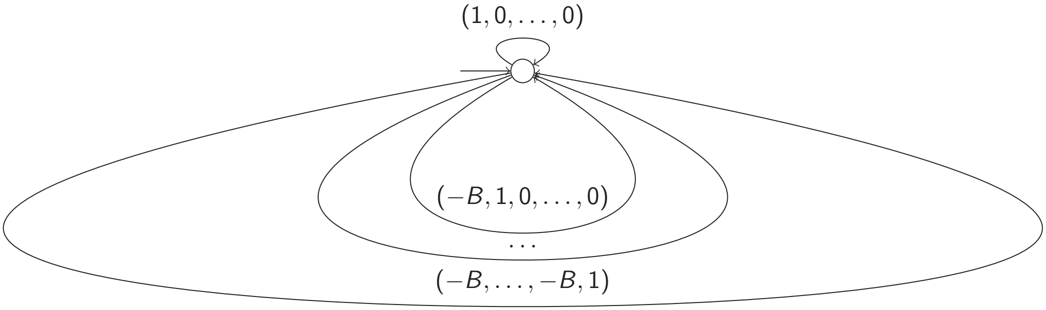

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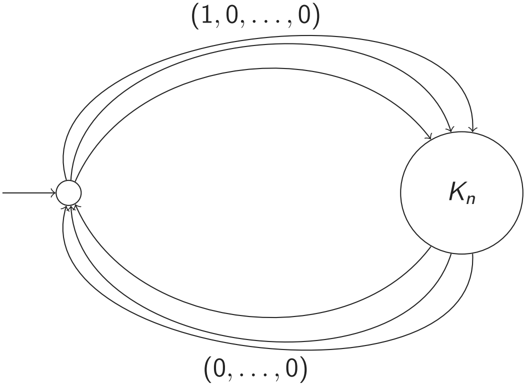
Lower bound: $(n-1)^2 < k$ case



$$\Lambda = (2B+1)B^{(n-1)^2-1}, \qquad \Pi = (n-1)^{(B-1)B^{(n-1)^2-1}}$$

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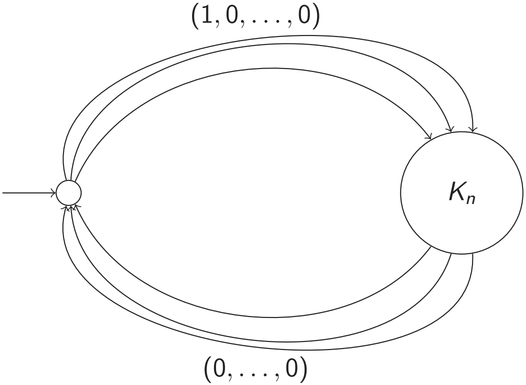

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How to find $\Pi(G)$

Central idea:

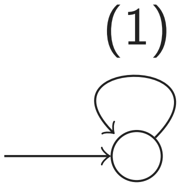
- Assume we have a Markov chain that allows us to sample paths uniformly at random
- Sample from it repeatedly
- Stop when you get the same path twice
- Output the square of the number of samples multiplied by $4/\pi^2$

Problems with the Markov chain method

Two main problems:

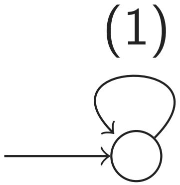
- How do we find a good Markov chain?
- Is there a divergence between the computed $\Pi(G)$ and the real $\Pi(G)$?

Markov chain example



Divergence: $\Pi' = \frac{\Pi}{\log(\Pi)}$

Markov chain example



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Open problems

A few central questions:

- How fast can we get the Markov chain to converge?
- Can we find general bounds on the divergence?
- Is there any general method that could approximate $\Pi(G)$ in $\Pi(G)^{o(1)}$
- Can we compute $\Lambda(G)$ in $o(\Lambda(G))$

Thank you for your attention